Lesson 2-4 Solving Equations with the Variable on Each Side

Many equations contain variables on each side. To solve these equations, FIRST use addition or subtraction to write an equivalent equation with all of the variables on one side.

[Traditionally, the variables (letters) are "moved" to the left and the constants (numbers) are "moved" to the right side of the equal sign.]

A. $8 + 5s = 7s - 2$

$8+5s-7s = 7s-7s-2$ (Subtract '7s' so variables are on the left.)

$8 + -2s = -2$ (Simplify)

$-8 \quad -8$ (Subtract '8' so constants are on the right.)

$\frac{-2s}{-2} = \frac{-10}{-2}$ (Simplify)

$s = 5$ (Multiply/divide)

(Simplify)

Check:

$8+5(5) = 7(5)-2$

$8+25 = 35-2$

$33 = 33$

B. $3 - \frac{1}{4}r = \frac{1}{2}r + \frac{3}{4}$

$3 - \frac{1}{4}r - \frac{1}{2}r = \frac{1}{2}r - \frac{1}{2}r + \frac{3}{4}$

$3 - \frac{1}{4}r - \frac{2}{4}r = \frac{3}{4}$

$3 - 3 - \frac{3}{4}r = \frac{3}{4} - 3$

$-\frac{3}{4}r = \frac{3}{4} - \frac{12}{4}$

$-\frac{3}{4}r = -\frac{9}{4}$

$-\frac{4}{3} \cdot -\frac{3}{4}r = -\frac{9}{4} \cdot -\frac{4}{3}$

$r = 3$

Check:

$3 - \frac{1}{4}(3) = \frac{1}{2}(3) + \frac{3}{4}$

$3 - \frac{3}{4}$	$\frac{3}{2} + \frac{3}{4}$
$2\frac{1}{4}$	$\frac{6}{4} + \frac{3}{4}$

$\frac{9}{4} = \frac{9}{4}$

Lesson 2-4 Solving Equations with the Variable on Each Side

Try these on your own:

PRACTICE:

1. $9f - 6 = 3f + 7$

2. $3 - 4q = 10q + 10$

3. $3k - 5 = 7k - 21$

4. $5x - 9 = {}^{-}3x + 7$

Solving Equations with Grouping Symbols

C. $\frac{1}{3}(18 + 12q) = 6(2q - 7)$

$6 + 4q = 12q - 42$	Original equation
$6 + 4q - 12q = 12q - 12q - 42$	Distributive Property [Think: $\frac{1}{3}\cdot 18 + \frac{1}{3}\cdot 12q$ AND $6\cdot 2q - 6\cdot 7$]
$6 - 8q = -42$	(Add/Subtract so variables are on the left.)
$6 - 6 - 8q = -42 - 6$	(Simplify)
$-8q = -48$	(Add/subtract so constants are on the right.)
$\frac{-8q}{-8} = \frac{-48}{-8}$	(Simplify)
$q = 6$	(Multiply/divide)
	(Simplify)

Check:

$\frac{1}{3}(18+12\cdot 6) = 6(2\cdot 6-7)$	
$\frac{1}{3}(18+72)$	$6(12-7)$
$\frac{1}{3}(90)$	$6(5)$
30	30

D. $6(3r - 4) = \frac{3}{8}(40r + 8)$ change problem

$18r - 24 = 15r + 3$

$18r - 15r - 24 = 15r - 15r + 3$

$3r - 24 = 3$

$3r - 24 + 24 = 3 + 24$

$\frac{3r}{3} = \frac{27}{3}$

$r = 9$

Check:

$6(3\cdot 9-4) = \frac{3}{8}(40\cdot 9+8)$	
$6(27-4)$	$\frac{3}{8}(360+8)$
$6(23)$	$\frac{3}{8}(368)$
138	138

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PRACTICE: Try these on your own

5. $3(a - 5) = -6$

6. $8 = 4(3c + 5)$

7. $4(2a - 1) = -10(a - 5)$

8. $2(w - 3) + 5 = 3(w - 1)$

No Solution and Identities

An equation that is true for every value of the variable is called an identity.
When solving an equation, if no variables remain and the resulting equation is true, the equation is an identity. (The solution will be all real numbers.) If there are no variables remaining and the resulting equation is false, then there is no solution.

F. $8(5c - 2) = 10(32 + 4c)$

$40c - 16 = 320 + 40c$

$-40c \quad -40c$

$-16 = 320$

False statement!

NO SOLUTION

G. $4(t + 20) = \frac{1}{5}(20t + 400)$

$4t + 80 = 4t + 80$

$-4t \quad -4t$

$80 = 80$

True Statement!

Infinitely many solutions
All Real Numbers (Identity)

H. $\frac{1}{7}(21c - 56) = 3\left(c - \frac{8}{3}\right)$

$3c - 8 = 3c - 8$

$-3c \quad -3c$

$-8 = -8$

Infinitely many
All Real Numbers
Identity

J. $2(4a + 8) = 3\left(\frac{8a}{3} - 10\right)$

$8a + 16 = 8a - 30$

$-8a \quad -8a$

$16 = -30$

no variables remain
false statement ($16 \neq -30$)
No Solution

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Try these or your own

PRACTICE:

9. $4(f - 2) = 4f$

10. $5h - 7 = 5(h - 2) + 3$

11. $3(1 + d) - 5 = 3d - 2$

* Preview of Lesson 2-7

12. $\frac{c+1}{8} = \frac{c}{4}$